

Basic Mathematics
Detail Course MTH104

Course Title: Basic Mathematics
Course No: MTH104
Nature of the Course: Theory
Semester: I

Full Marks: 60 + 40
Pass Marks: 24 + 8 + 8
Credit Hrs: 3

Course Description:

This course familiarizes students with functions, limits, continuity, differentiation, integration of function of one variable, logarithmic, exponential, applications of derivative and antiderivatives, differential equations, partial derivatives.

Course Objectives:

1. Students will be able to understand and formulate real world problems into mathematical statements.
2. Students will be able to develop solutions to mathematical problems at the level appropriate to the course.
3. Students will be able to describe or demonstrate mathematical solutions either numerically or graphically.

Unit 1 Functions and their graphs

[5 Hrs.]

Definition, domain range, Graphs of functions, Representing a function numerically, the vertical line test for a function, Piecewise defined functions, Increasing and decreasing functions, Even and odd function, Common functions: linear, power, polynomial, rational functions
All worked out examples of 1.1.

Exercises 1.1: 1-8, 15, 18, 23, 25, 26.

1.2: Combining functions: Shifting and Scaling graphs

Sums, differences, products and quotients, Composite functions, Shifting a graph of a function.

Worked out examples: 1-5

Exercises 1.2: 1-8.

1.4: Graphing with calculator and computers (desmos may be easy) to plot the graph of the functions (some of the functions):

$$y = x, y = x^2, y = \frac{1}{1-x}, y = \sin x, y = \cos x, y = \sin 100x$$

1.5: Exponential functions: Definition, Exponential behavior, Exponential growth and decay.

Worked out examples: 1-4.

Exercises 1.5: 29-33

1.6: Inverse Functions and Logarithms

Worked out examples: 1 - 4, 6, 7.

Exercises 1.5: 79 - 81

2.1: Rate of change and tangent to curves.

Worked out examples: 1-5.

Exercises 2.1: 1, 3, 6, 7, 9, 15, 17.

Unit 2. Limits and continuity

[3 Hrs.]

2.2 Limit of a Function and Limit Laws

Limits of function values, The limit laws, Eliminating zero denominators algebraically, The Sandwich theorem(no proof).

Worked out examples: 1-11

2.3 The Precise Definition of a Limit

Definition of limit

Worked out examples: 1-5

One sided limit: Worked out Examples 1-4

2.5 Continuity

Worked out examples: 2, 3

Intermediate Value Theorem for Continuous Functions

Worked out examples: 11, 12

2.6 Limits Involving Infinity; Asymptotes of Graphs

Worked out examples 1, 2, 3

Horizontal Asymptotes

Worked out examples: 4-9

Oblique asymptotes

Worked out examples: 10-14

Vertical asymptotes

Worked out examples: 15-19.

Some related problems

Unit 3. Differentiation

[3 Hrs.]

3.1 Tangents and the Derivative at a Point

Finding a Tangent to the Graph of a Function

Rates of Change: Derivative at a point

Worked out Examples: 1, 2

Exercises 3.1: 5-8, 11, 12, 13, 23, 24, 25

3.2 The Derivative as a function

Worked out Examples: 4, 5

Differentiable Functions are continuous

3.4 The Derivative as a rate of change

Worked out Examples: 1-7 Ideas of derivatives of trigonometric, inverse trigonometric, logarithm, exponential functions and ideas of chain rules.

3.10 Related rates

Worked out Examples: 1-6.

Unit 4 Application of Derivative

[5 Hrs.]

4.1 Extreme values of functions: Introduction

Worked out examples: 1-4

Exercise 4.1: 21, 22, 23, 31, 32

4.2 The mean value theorem

Rolle's Theorem(no proof), Lagrange mean value theorem(no proof)

Worked out examples: 1-4

4.3 Monotonic functions and the first derivative test

Increasing functions and decreasing Functions

Worked out examples: 1, 2, 3

4.4 Concavity and curve sketching

Worked out examples: 1-9

4.5 Indeterminate Forms and LHpitals Rule

Indeterminate form, LHpitals rule

All worked out examples

Exercises 4.5: 1-7, 13, 15. 4.6 Applied optimization

Worked out examples: 1-5

4.7 Newton's method.

Worked out examples: 1, 2

Examples 4.7: 1-4

Unit 5 Integration

[5 Hrs.]

4.8 Antiderivatives

Worked out examples: 1, 2, 3

5.1 Area and Estimating with Finite Sums

Area

worked out examples: 1-4

Exercises 5.1: 1-4

5.2 Sigma notation and limits of finite sums

Worked out examples: 1-5

5.3 The definite integral

Worked out example: 4, 5

5.4 The fundamental theorem of calculus

Mean value theorem for definite integrals, Fundamental theorem of calculus Part 1 and 2 (no proof), The net change theorem

Worked out examples: 2-7

5.5 Indefinite integral and substitution method:

All worked out examples

5.6 Area between the curves

Worked out examples: 4, 5, 6, 7

Exercises 5.6 : 63-66

Unit 6 Application of Definite Integrals

[5 Hrs.]

6.1 Volumes using cylindrical shells

Worked out examples: 1-10

6.2 Volumes using cross-sections

Worked out examples: 2, 3

6.3 Arc length

Worked out examples: 1, 2 3, 4, 5

6.4 Areas of surfaces of revolution

Worked out examples: 1, 2

Unit 7 Techniques of Integrations

[5 Hrs.]

Review of integration by parts, trigonometric substitutions, integration of rational functions by partial fractions. Computer algebra system (Maple)

8.6 Numerical Integration

Numerical Integration

Simpsons Rule: Approximations Using Parabolas

Error Analysis

Worked out examples:1-6

Exercises 8.6: 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 19, 21.

8.7 Improper integrals

Worked out examples: 1-9

Unit 8 First Order Differential Equations

[4 Hrs.]

9.1 Solutions, Slope Fields, and Eulers Method

General first order differential equations and solutions

Worked out examples: 1, 2.

Slope Fields: Viewing Solution Curves

Eulers Method

Worked out examples: 3, 4

Exercises 9.1: 11, 12, 13

9.2 First order linear equation

Worked out examples 1, 2, 3

Exercises 9.2: 1-10, 15-21

9.3 Applications

Motion with resistance proportional to velocity
7.2 Exponential change.
Worked out Examples: 1, 2, 3, 4, 5.
9.4 Graphical solutions of autonomous equations
Example worked out: 1

Unit 9. Infinite Sequence and Series

[5 Hrs.]

10.1. Sequences

Worked out Examples: 1-11

Exercises 10.1: 1,2,3,7,8,13,16,27-32 Infinite series

Worked out examples: 1-10

Related problems from exercise 10.2

Ideas of Integral test, comparison test: worked out examples, Alternating series, absolute and conditional convergence, with at least one worked out examples.

10.7. Power series

Worked out examples 1-6.

10.8. Taylor and Maclaurin series

Exercises 10. 8: 1, 2, 3, 4, 7, 9, 11, 12

Unit 10 Partial Derivatives

[5 Hrs.]

14.1. Functions of several variables

Worked out examples: 1, 2, 3, 4

14.2 Limits and continuity in higher dimensions

Worked out Examples: 1-6

Exercises: 1, 2, 3, 4, 5, 6, 13, 14

14.3. Partial derivatives

Worked out examples: 1-10

Examples 14.3: 1-18

14.5. Chain rule

Worked out examples: 1-6

14.5. Directional derivative

Worked out examples: 1-5

14.6. Tangent planes and differentials

Worked out examples: 1-4

14.7. Extreme values and saddle points

Worked out examples: 1-5

Exercises 14.7: 1-7

Text/ Reference Books:

1. M. D. Weir, J. Hass, F. R. Giordano: *Thomas Calculus*, Twelfth Edition, 2009, Pearson.

Guidelines to the question setter:

1. Questions must include every unit.
2. There will be two groups, namely, **Group A** and **Group B**.
3. **Group A**, there must be one choice among three questions.
4. In **Group B**, there must be one choice among 9 questions.
5. Questions must be creative and should be appropriate to the allocated time.

On the basis of the guidelines mentioned, we enclose one set of model question for Calculus (MTH104)

TRIBHUVAN UNIVERSITY
INFORMATION TECHNOLOGY ,
2078

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Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt **All** the questions.

Group A

[2 × 10 = 20]

Attempt any two questions

1. (a) In 2000, 100 is invested in a savings account, where it grows by accruing interest that is compounded annually (once a year) at an interest rate of 5.5%. Assuming no additional funds are deposited to the account and no money is withdrawn, give a formula for a function describing the amount A in the account after x years have elapsed. [5]
- (b) Define when the function $f(x)$ is odd and even. Also, define when a function $f(x)$ is increasing and decreasing? If $y = x^2$ is a given function then determine the interval in which the function is increasing and decreasing and draw the graph of the given function. [1 + 1 + 1 + 2]
2. A rock breaks loose from the top of a tall cliff [3 + 3 + 4]
 - (a) Find average speed during the first 2 sec of fall.
 - (b) What is its average speed during the 1sec interval between second 1 and second 2?
 - (c) Find the speed of the falling rock at $t = 1$ and $t = 2$.
3. (a) Find the positive root of the equation [3]

$$f(x) = x^2 - 2 = 0$$

- (b) Find the Taylor series and the Taylor polynomials generated by $f(x) = e^x$ at $x = 0$. [3]
- (c) Use the Trapezoidal Rule with $n = 4$ to estimate $\int_1^2 x^2 dx$. Compare the estimate with the exact value. [4]

Group B

Attempt any 8 questions

[8 × 5 = 40]

4. Define horizontal asymptote to a curve $y = f(x)$. Find the horizontal asymptote to the curve

$$f(x) = \frac{5x^2 + 8x - 3}{3x^2 + 2}$$

and draw the curve.

5. (a) Find the slope of the curve $y = 1/x$ at any point $x = a, a \neq 0$. What is the slope at the point $x = -1$?
 (b) Where does the slope equal $-1/4$?
 (c) What happens to the tangent to the curve at the point $(a, 1/a)$ as a changes?
6. Water runs into a conical tank at the rate $9ft^3/minutes$. The tank stands point down and has a height of $10ft$ and a base radius of $5ft$. How fast is the water level rising when the water is $6ft$ deep?
7. Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-2, 3]$.
8. Find the area between the curves

$$y = x^2 - 2 \text{ and } y = 2$$

9. A pyramid $3m$ high has a square base that is 3 m on a side. The cross section of the pyramid perpendicular to the altitude xm down from the vertex is a square x m on a side. Find the volume of the pyramid.
10. Draw a phase line for the equation

$$\frac{dy}{dx} = (y + 1)(y - 2)$$

and use it to sketch solutions to the equation.

11. Find the second order derivative

$$\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y \partial x}$$

of

$$f(x, y) = x \cos y + ye^x$$

12. Find the local extreme values of the function

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4.$$