Orientation of Mathematics II (MTH 163)]
For B.Sc. CSIT
Date: 2074/ 09/ 14 and 2074/ 09/15
By

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and

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Course Title: Mathematics II — Full Marks: 80
Course No: MTH 163 — Pass Marks: 32
Nature of the course: Theory and Practice — Credit Hrs.: 3
Semester: II

Course Contents:

1. Linear Equations in Linear Algebra: System of Linear equations, Row reduction and Echelon forms vector equations The matrix equations A **x =b** Applications of linear system, Linear independence. **5 Hours.**

2. Transformation: Introduction to linear transformations, the matrix of a linear transformation, Linear models in business, science and engineering.. **4 Hours.**

3. Matrix Algebra: Matrix operations The inverse of a matrix, Characterizations of invertible matrices, Partitioned matrices Matrix factorization, The Leontief input output model subspace of $\mathbb{R}^n$, Dimension and rank **5 Hours.**

4. Determinants: Introduction, Properties, Cramer's rule, Volume and linear transformations. **4 Hours.**

5. Vector Spaces: Vector spaces and subspaces, Null spaces, Column spaces, and Linear transformations, Linearly independents sets: Coordinate systems. **5 Hours.**

6. Vector Space Continued: Dimension of a vector space and rank, Change of basis, Applications to difference equations, Applications to Markov chains. **4 Hours.**

7. Eigenvalues and Eigenvectors: Eigenvectors and Eigenvalues, The characteristic equations, Diagonalization, Eigenvectors and linear transformations, Complex eigenvalues Discrete dynamical systems, Applications to differential equations. **5 Hours.**

8. Orthogonality and Least Squares: Inner Product, Length, and orthogonality, Orthogonal sets Orthogonal projections, The Gram Schmidt process, Least squares problems, Application to linear models, Inner product spaces Applications of inner product spaces. **5 Hours.**

9. Groups and subgroups: Binary operations, Groups, Subgroups, Cyclic Groups. **5 Hours.**

10. Rings and Fields: Rings and Fields, Integral domains. **4 Hours.**

Text Books:

1. Linear Algebra and its Applications, David C. Lay, 4^{th} Edition, Pearson Addison Wesley.
2. Linear Algebra and its Applications, Gilbert Strang, 4^{th} Edition, CENGAGE Learning
3. A First course in Abstract Algebra, John B. Fraleigh, 7^{th} Edition, Pearson

Detail Course:

1. Linear Equations in Linear Algebra:

Definition of linear equation, Consistent and inconsistent; Matrix notation, Solving a linear system, Existence and uniqueness questions, Related problems(Ex. 1.1(no. 1-22, 29-32)), Definition of echelon form and reduce row echelon form, Examples, Pivot positions, Related problems(EX. 1.2 (no. 1-14, 17-20)) Vectors in $\mathbb{R}^2$, Geometric description of $\mathbb{R}^2$, Vectors in $\mathbb{R}^3$, Geometric description of $\mathbb{R}^3$, Vectors in $\mathbb{R}^n$, Linear combinations, A geometric description of span **v**,
Ex.1.3 (no. 1-18) The matrix equation A **x=b**, Ex. 1.4 (no. 1-15), Application of linear systems, A homogeneous system in economics, Linear independence, Linear independence of matrix columns, Sets of one or two vectors and related problems(Ex. 1.7 (no. 1-20)). **5 Hours.**

2. Transformation :

Introduction to linear transformations, Matrix transformations, Linear transformations, Related problems(Ex. 1.8 (no 1- 19)). , The matrix of a linear transformation, Geometric linear transformation of $\mathbb{R}^2$, One to one and onto mapping, Related problems(Ex. 1.9 (no. 1-10)), Linear models in business, science, and engineering (example 1, 2, and 3). **4 Hours.**

3. Matrix Algebra:

Matrix operations, Sum and scalar multiples, Matrix multiplication, Properties of matrix multiplication, The transpose of a matrix, Related theorems and related problems(Ex. 2.1 (no. 1-12, 16, 17, 27,28)), The inverse of a matrix, Invertible, Singular and non singular matrix, Elementary matrices, Characterizations of invertible matrices, Related problems(Ex. 2.2 (no. 1-7)), Partitioned matrices, Addition and scalar multiplication multiplication of partitioned matrices, Multiplication of partitioned matrices, Inverses of partitioned matrices, Related problems(Ex. 2.3 (no. 1-10), Ex. 2.4 (no. 1-10)), Matrix factorizations, The LU factorization, Related problems(Ex. 2.5 (no. 1-15)), The Leontief input- output model (example 1 and 2), Subspaces of $\mathbb{R}^n$, Column space and null space of a matrix, Related problems(Ex. 2.8 (no. 1-20, 23-26)), Dimension and rank, Coordinate systems, The dimension of subspace, The rank theorem (no proof), The basis theorem (no proof) and invertible matrix theorem (no Proof), Ex. 2.9 (no. 1-7) **5 Hours.**

4. Determinants:

Introduction to determinants, Properties of determinants, Determinants and matrix products, Related problems(Ex. 3.1 (no. 1-38) Ex. 3.2 (no. 1-26), Cramer's rule, Volume,and linear transformations, An inverse formula and Related problems(Ex. 3.3 (no. 1-22)), Determinants as area or volume(example 4), Linear transformation. **4 Hours.**

5. Vector Spaces:

Vector spaces, Subspaces, Zero subspaces, A subspace spanned by a set (Theorem 1 (no proof)), The null space of a matrix, Theorem 2, The column space of a matrix, The contrast between nul A and col A (example 5, 6, 7), Kernel and range of a linear transformation(example 8, 9), Related problems(Ex. 4.1 (no. 1-18) (Ex.4.2 (no. 1-24), Linearly independent sets; Bases, Examples, The spanning theorem (no proof), Bases for nul A and col A, Theorem 6 (no proof), Coordinate systems, The unique representation theorem, The coordinate mapping, Related problems(Ex. 4.3 (no. 1-20) Ex. 4.4 (no. 1-14)). **5 Hours.**

6. Vector Space Continued:

The dimension of a vector space, Theorem 9, 10 (no proof, concept only), Subspace of a finite dimensional space, Theorem 11,12 (no proof), The dimension of nul A and col A, Related problems(EX. 4.5 (no. 1-17)), The row space, Theorem 13 (no proof), The rank theorem (no proof), The invertible matrix theorem (No proof), Related problems(Ex. 4.6 (no. 1-7)), Change of basis, Theorem 15 (no proof), Change of basis in $\mathbb{R}^n$, Application to difference equations (example 1, 2, 4), Related problems(Ex. 4.7 (no. 1-10)), Applications to Markov chains(example 1, 2) . **4 Hours.**

7. Eigenvalues and Eigenvectors:

Eigenvectors and eigenvalues, Theorem 1, 2, Related problems(Ex. 5.1 (no. 1-20)), The characteristic equation, Similarity, Application to dynamical systems(example 5), Related problems(Ex. 5.2 (no. 1-14) Ex. 5.3 (no. 1-18)), Diagonalization, The diagonalization theorem (no proof), Diagonalizing matrices (theorem 6), Eigenvectors and linear transformations, The matrix of a linear transformation, Linear transformations on $\mathbb{R}^n$, Diagonal matrix representation (no Proof), Complex eigenvalues, Real and imaginary parts of vectors, Related problems(Ex.5.4 (no. 1-7) ((Ex. 5.5 (no. 1-20)), Discrete dynamical systems (example 1,2, 3, 4), Applications to differential equations (example 1, 2) **5 Hours.**

8. Orthogonality and Least Squares:

The inner product,theorem 1, The length of a vector, distance, Orthogonal vectors, Orthogonal complements, The pythagorean theorem, Theorem 3 (no Proof), Angles in $\mathbb{R}^2$ and $\mathbb{R}^3$, Related problems(Ex. 6.1 (no. 1-18), Orthogonal sets, Theorem 4, Orthogonal basis, Theorem 5, An orthogonal projection, Orthonormal sets, Theorem 6, Theorem 7(no proof), Related problems(Ex. 6.2 ((no. 1-22)), Theorem 8 (no proof),

The Gram- Schmidt process, Theorem 11 (no proof), Least square problems, Solution of the general least square problem, Theorem 13(no proof), Theorem 14 (no proof), Theorem 15 (no proof), Related problems(Ex. 6.3 (no. 1-18) Ex. 6.4 (no. 1-8)), Application to linear models (example 1, 2, 3),

Inner product spaces, Length, Distance, and orthogonality, Cauchy -Schwarz inequality, Related problems(Ex. 6.5 (no. 1-14)(Ex. 6.7 (no. 1-6)), Application of inner product spaces (example 1, 2) **5 Hours.**

9. Groups and subgroups:

Sets, Introduction and examples, Complex numbers, Multiplication of complex numbers, Related problems, Binary operations, Definitions and examples, Commutative, Associative, Related problems, Isomorphic binary structures, Definition and examples, Uniqueness of identity element (no proof), Exercises 3 (1 - 10), Definition of group and examples, Abelian group, Elementary properties of groups, Finite groups and group table, Exercises 4 (1- 6), Order of a group, Subgroups and examples, Proper and improper subgroup, Trivial and non trivial subgroup, Examples, Theorem 5.14 (no proof), Generator of a group, Definition of cyclic group and examples, Theorem 5.17 (no proof), Exercises 5 (1- 12, 14- 19, 21,26, 27). **5 Hours.**

10. Rings and Fields:

Definition of ring and basic properties, Examples, Theorem 18.8, Exercises 18 (1- 12), Divisors of zero and examples, Integral domain, Examples, Definition of field, Theorem 19.9, Theorem 19.11 (no proof), Corollary 19.12 (no Proof), Exercises 19 (1- 4). **4 Hours.**

Notes to the Question Setter:

1. All units are equally important, so questions must be made from every units with equal marks as far as possible.
2. In group A, a student can answer any three questions selecting among 4 long questions, each carrying 10 marks. These 4 questions should cover all the 10 units as far as possible.
3. In group B, a student can answer any 10 questions selecting among 11 short questions, each carrying 5 marks. Among these 11 questions, 10 are made from each unit and remaining one question may be made from any one of the 10 units.
4. Questions must be creative and should fit to the allocated time.

Model Question
Tribhuvan University
Institute of Science and Technology

Bachelor Level/ First Year/Second Semester/ Science — Full Mark:80
Computer Science and Technology (MTH 163) — Pass Marks: 32
Mathematics II — Time: 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

Group A ($10 \times 3 = 30$)

Attept any **THREE** questions.

1. What is pivot position? Apply elementary row operation to transform the following matrix first into echelon form and then into reduced echelon form:
$$\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}.$$ [2+8]

2. Define linear transformation with an example.Check the following transformation is linear or not? T: $\mathbb{R}^2 \to \mathbb{R}^2$ be defined by
T(x,y) = (x, 2y).Also, let T(x,y)= (3x+y, 5x+7y, x+3y). Show that T is a one- to-one linear transformation. Does T maps $\mathbb{R}^2$ onto $\mathbb{R}^3$?
. [3+ 2+5]

3. Find the LU factorization of
$$\begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}.$$ [10]

4. Find a least square solution of the inconsistent system A**x= b** for
A= $\begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$, **b**= $\begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$. [10]

Group B ($10 \times 5 = 50$)

Attept any **TEN** questions.

5. Compute **u+ v, u-2v and 2u+v** where **u**= $\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$. **v**= $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$.
. [5]

6. Let A= $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, and define T: $\mathbb{R}^2 \to \mathbb{R}^2$ be defined by
T(**x**) = A**x**, find the image under T of **u**= $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ and **v**= $\begin{bmatrix} a \\ b \end{bmatrix}$.
. [5]

7. Let A= $\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value(s) of k, if any, will make AB= BA?

8. Compute det A, where A= $\begin{bmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{bmatrix}$. [5]

9. Let H be the set of all vectors of the form $\begin{bmatrix} 2t \\ 0 \\ -t \end{bmatrix}$. Show that H is a subspace of $\mathbb{R}^3$ [5]

10. Find basis and the dimension of the subspace
$H = \left\{ \begin{bmatrix} \text{s-2t} \\ \text{s+t} \\ 3t \end{bmatrix}, s, t \in \mathbb{R} \right\}$. [5]

11. Find the eigenvalues and eigenvectors of A= $\begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$ [5]

12. Define orthogonal set. Show that $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is an orthogonal set, where $\mathbf{u_1}$= $\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{u_2}$= $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{u_3}$= $\begin{bmatrix} -\frac{1}{2} \\ -2 \\ \frac{7}{2} \end{bmatrix}$ [5]

13. Let $W = span\{\mathbf{x_1}, \mathbf{x_2}\}$, where $\mathbf{x_1} = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ and $\mathbf{x_2} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ Construct an orthogonal basis $\{\mathbf{v_1}, \mathbf{v_2}\}$ for W. [5]

14. Let * be defined on $\mathbb{Q}^+$ by a* b= $\frac{ab}{2}$. Then show that $\mathbb{Q}^+$ forms a group.
. [5]

15. Define ring with an example. Compute the product in the given ring (12)(16) in $\mathbb{Z}_{15}$ [5]